Radiometry in the Submillimeter Region Using the Interferometric Modulator*

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Summary—Radiometry in the submillimeter and far infrared regions involves problems of a type not encountered in the centimeter region which require solutions using techniques different from those used in centimeter-wavelength radiometry. The nonlinear variation of the magnitude of the black-body radiation spectral density with temperature and wavelength, the limitation of antenna beamwidth by factors connected with the size of the noncoherent detector and the antenna focal length (rather than by diffraction effects and the antenna aperture) and the heavy absorption of submillimeter radiation by atmospheric water vapor are typical of the problems normally not encountered in centimeter radiometry.

The unavailability of microwave techniques (*i.e.*, waveguides, coherent receivers, etc.) makes necessary the use of quasi-optical techniques in this wavelength region. The interferometric modulator, which has already been used in far infrared spectrometers, is proposed in this paper as the major component of a practical submillimeter radiometer. Its use as the wave-number-selection device in a radiometer is analyzed and estimates are obtained for the sensitivity of this submillimeter radiometer. It is estimated that a 0.2° minimum detectable temperature differential is achievable with this radiometer.

Also discussed are the effects of atmospheric water vapor absorption and the sensitivity of a number of different types of radiation detectors suitable for use in the submillimeter-wavelength region.

Introduction

A RADIOMETER, whether at centimeter or submillimeter wavelengths, consists of a sensitive frequency-selective receiver connected to a directional energy collector (or antenna) which is pointed towards the radiation source. A possible source of radiation is a black body at some temperature T. It is customary to characterize a radiation source by its equivalent black-body temperature T in a given wavelength region, whether the actual source is a black body or not. The purpose of a radiometer is the detection or the measurement of the radiation power in a given wavelength region in a given direction.

Radiometry in the submillimeter-wavelength region differs from radiometry in the centimeter region in several respects.

1) Due to the fact that the Rayleigh-Jean's approximation to the exact Planck radiation law is no longer valid for black-body radiation in this wavelength range [1], [2] the relation between noise power and temperature is no longer linear.

2) The lack of sensitive receivers such as the superheterodyne receivers used in the centimeter-wavelength region requires the use of quasi-optical techniques for the selection (in terms of wavelength or frequency) and detection of the desired radiation. Such devices as thermal radiation detectors and interferometric modulators (to be described) must be used in constructing a receiver for operation at these wavelengths.

3) The size of a thermal radiation detector (usually on the order of a square centimeter in area) for use at submillimeter wavelengths is much larger than the diffraction-limited focus spot size ($\approx\lambda^2$) produced by the optical mirror system which is used as the antenna of the submillimeter radiometer. Because of this, the beamwidth of the submillimeter radiometer is determined by geometrical considerations associated with the large size of the thermal detector imaged upon the receiver input (or upon the aperture size of the receiver input, whichever is smaller), rather than by the diffraction limit as determined by the diameter of the collecting mirror. This is illustrated in Fig. 1.

The particular problems of the submillimeter radiometer, such as the relation between the minimum detectable temperature change and the minimum detectable power change, the interferometric submillimeter re-

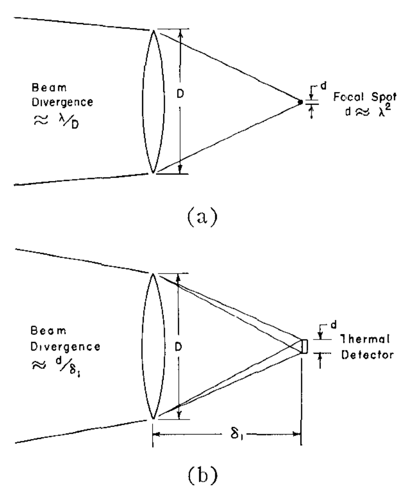

Fig. 1—Comparison of radiation-collector beamwidths. (a) Diffraction-limited optical system. (b) Geometrically-limited optical system.

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ceiver, and the calculated and measured sensitivity, are discussed in the following sections. It is concluded from this study that a submillimeter radiometer with a 0.2°K minimum detectable temperature-change sensitivity is achievable by this technique [1].

Minimum Detectable Temperature Difference

Consider the minimum variation in the equivalent black-body temperature T of a source which can be detected by a radiometer equipped with a receiver which can detect a minimum change in its input power of $[\Delta P \text{ min}]$. It can be shown [1] that

$$[\Delta T \text{ min}] = \frac{\eta\Gamma}{ck[\Delta\nu]}[\Delta P \text{ min}]°\text{K} \qquad (1)$$

where

T = equivalent black-body temperature of the source
c = speed of light
k = Boltzmann's constanti
$[\Delta\nu]$ = bandwidth of the radometer receiver in wave number units[1]
η = a factor depending upon λ and T which expresses the correction due to the use of the exact form of the radiation law. η is plotted in Fig. 2 vs λ and T. η is equal to unity at centimeter wavelengths.
Γ = a factor expressing the dependence of $[\Delta T \text{ min}]$ upon the size, source of, wavelength of observation and the parameters of the radiometer optical system.

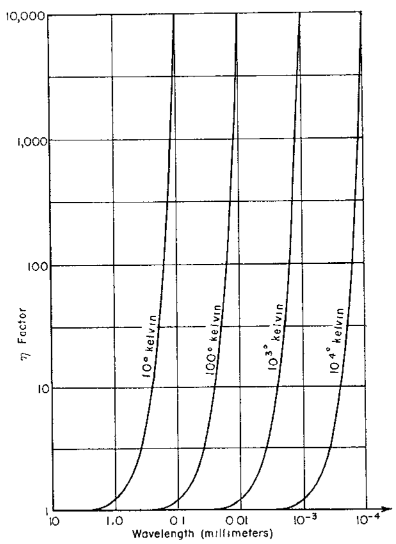

Fig. 2—η factor vs temperature and wavelength.

[1] The wave number unit is a measure of frequency given by $\nu = 1/\lambda = f/c$ where λ = free-space wavelength of the radiation in question and c = velocity of light.

The equations for η and Γ are [1]

$$\eta = \left[\frac{\lambda kT}{ch}\right]^2 \frac{[\epsilon^{ch/\lambda kT} - 1]^2}{\epsilon^{ch/\lambda kT}} \qquad (2)$$

$$\Gamma \begin{cases} = \dfrac{\lambda^2\delta_1^2}{2sA_m} = \dfrac{2\lambda^2 F_0^2}{\pi s} \\ \quad \text{for } [\Delta\Omega]_s > \text{beamwidth at submillimeter wavelengths} \\ = 1 \text{ (unity)} \\ \quad \text{for } [\Delta\Omega]_s > \text{beamwidth at centimeter wavelengths} \\ = \dfrac{\lambda^2}{2A_m[\Delta\Omega]_s} \\ \quad \text{for } [\Delta\Omega]_s < \text{beamwidth at submillimeter wavelengths} \\ = \dfrac{\lambda^2}{A_e[\Delta\Omega]_s} \\ \quad \text{for } [\Delta\Omega]_s < \text{beamwidth at centimeter wavelengths} \end{cases} \qquad (3)$$

where

h = Planck's constant
A_e = effective antenna aperture (for the case of a linearly polarized centimeter-wavelength antenna)
A_m = aperture area (geometrical) of collecting mirror (submillimeter radiometer)
$[\Delta\Omega]_s$ = solid angle subtended by the source when viewed from the radiometer
s = input aperture area of the submillimeter receiver or the image size of the detector at the input (whichever is smaller)
δ_1 = focal length of the submillimeter-radiometer collecting mirror (antenna)
F_0 = f-stop number[2] of the collecting mirror system (or of the submillimeter receiver if its f stop is *numerically* greater than that of the antenna, in which case F_r is substituted for F_0).

We can also relate $[\Delta T \text{ min}]$ to the minimum detectable change in the spectral density $[E(\nu) \text{ min}]$ of the incoming radiation by letting $[E(\nu) \text{ min}] = [\Delta P \text{ min}]/[\Delta\nu]$ in (1).

Note that in the case of the centimeter radiometer, looking at a large source ($[\Delta\Omega]_s >$ than the beamwidth), the sensitivity is not a function of the antenna area as long as the antenna is not made so small that the beamwidth becomes larger than the source. However, in the submillimeter case, the beamwidth is not determined by the antenna area, and the temperature sensitivity can be increased (theoretically) by making F_0 smaller (by

[2] The f-stop number of an optical element is the ratio of its focal length to its diameter.

making the antenna area larger and by using larger mirrors within the submillimeter receiver). The beamwidth of the submillimeter radiometer is given by

$$\sqrt{\frac{4s}{\pi}} \Big/ \delta_1 \text{ as compared with } 1.22\lambda \Big/ \sqrt{\frac{4A_e}{\pi}}$$

for the centimeter radiometer.

The Analysis of the Interferometric Modulator as Used in a Submillimeter-Radiometer Receiver

The interferometric modulator is a quasi-optical device used to select a narrow spectral range from the incoming radiation. It does this by modulating the incoming radiation at an audio frequency which is proportional to the frequency of the incoming radiation. Its operation is described in this section.

There are two elementary types of interferometric modulators: aperiodic and periodic, depending upon the mode of operation. However, both types operate upon the basic principle of dividing the incoming radiation into two or more beams, half of the radiation being sent along a path which is γ cm longer than the path followed by the other half. When the radiation is recombined at a square-law radiation detector, interference results and the output power depends upon the value of γ and the wave number v of the input radiation. The use of the aperiodic modulator has been considered elsewhere [1], [3]–[11]. Since the use of the periodic interferometric modulator makes possible the use of real-time filtering and correlation techniques, it is used in the radiometer described in this paper.

The physical form of the interferometric modulator is usually either a form of the Michelson interferometer or else is a lamellar grating of variable groove depth. In the grating type of interferometric modulator (see Fig. 3) the radiation is reflected from a lamellar grating in which the groove depth is varied by moving a set of movable lamella between a set of fixed lamella. The set of movable lamella is attached to a frame which is driven by a reciprocating mechanism. The groove depth, denoted by $x(t)$ is varied linearly from $x=0$ to $x=X_m$, the maximum groove depth, and back again to $x=0$ once every $2T_m$ sec. The radiation incident upon the interferometric modulator is reflected by the fixed and movable lamella and is collimated by a collecting mirror. The path-length difference between the light reflected from the fixed and movable lamellae is given by $\gamma(t)=2x(t)/\cos\xi$ where ξ is the angle of incidence and reflection measured from the normal. Whenever $\gamma(t)=n\lambda/2$ and n is an odd integer, destructive interference occurs and the modulator output is reduced to zero. When $\gamma(t)=n\lambda/2$ and $n=0$ or an even integer, constructive interference occurs and the output is a maximum (unity transmission). Therefore, as the groove depth varies linearly from 0 to X_m, the collected radiation power of wavelength λ at the output of the modulator is modulated at an audio frequency given by $f_\lambda=\gamma_m/\lambda T_m$. The voltage output of a radiation detector placed at this point due to the radiation at wavelength λ will also be modulated at $f_\lambda=\gamma_m/\lambda T_m$. It should be noticed that $\gamma_m/T_m=v$ is the velocity of the path-length change where γ_m is the maximum path-length difference. The operation of an interferometric modulator using a Michelson interferometer is entirely similar to the operation of the grating interferometric modulator except that $\gamma\equiv 2x$ where x is now the displacement of the movable mirror (*i.e.*, the angle $\xi=0$). A sketch of this form of modulator mechanism is shown in Fig. 4. Fig. 5 is a plot of the path-length variation γ of either type of periodic interferometric modulator as a function of time. If the path-length difference γ were to be varied at an even rate from zero to infinity, then for a continuous radia-

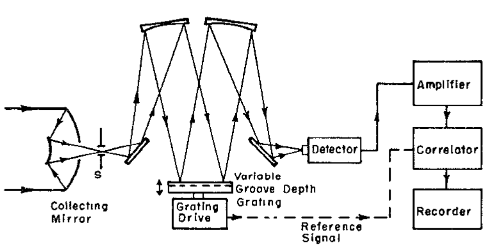

Fig. 3—Lamellar grating type of an interferometric radiometer.

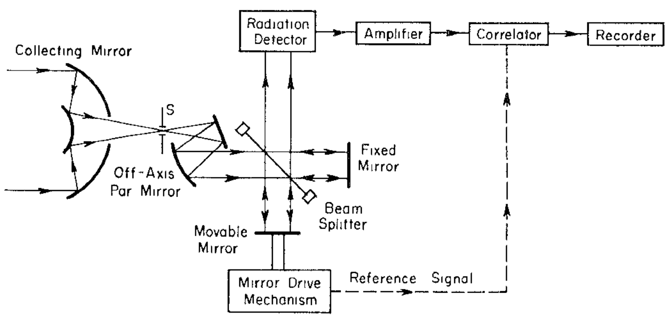

Fig. 4—Michelson-type interferometric modulator.

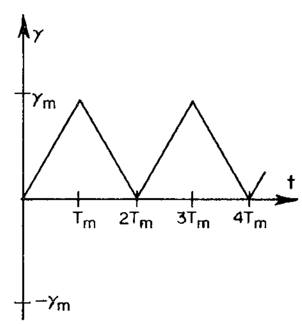

Fig. 5—Change of path-length difference γ with time of the periodic modulator.

tion input spectrum the radiation detector output would be continuously modulated and would have a continuous modulation spectrum. However, in the case of the periodic interferometric modulator where γ varies from 0 to γ_m and back again, only when the wavelength λ of the radiation is such that $\lambda=\lambda_n=2\gamma_m/n$ where n is an integer will the output from the radiation detector be a continuous cosine wave (at frequency $f_n=\gamma_m/T_m\lambda=\gamma_m\nu/T_m$). If $\lambda\not\equiv 2\gamma_m/n$, then discontinuities are introduced in the detector output every $2T_m$ sec when $\gamma(t)=\gamma_m$. This discontinuity is necessary for the correct operation of the periodic interferometric modulator as shown in the following considerations.

Let the input to the interferometric modulator consist of a continuous spectrum of radiation in the far infrared and the submillimeter-wavelength regions. The output radiation of the modulator will also have this same continuous spectrum of radiation, but the radiation at each wavelength λ will be modulated (continuously or semi-continuously) at an audio frequency given by $f_\lambda=\gamma_m/T_m\lambda=\gamma_m\nu/T_m$. We detect this total radiation and correlate the output of the radiation detector with a cosine wave at some frequency $f_{n0}=\gamma_m/T_m\lambda_{n0}$ where f_{n0} is such that $\lambda_{n0}=2\gamma_m/n'$ where n' is some particular integer. Since the Fourier integral of a periodic time function $\cos 2\pi f_0 t$ is zero unless $f=f_0$, in a modulator where $\gamma_m\rightarrow\infty$ (*i.e.*, no reversal of the grating motion and no discontinuity in modulation), the correlator would produce a dc output only for $\lambda\equiv\lambda_{n0}$ (for a radiation detector output frequency of exactly f_{n0}). This would not be very practical since only an infinitesimal amount of the radiation power is concentrated exactly at λ_{n0}.

However, since in the practical periodic interferometric modulator the modulation action is discontinuous once every $2T_m$ sec (Fig. 6) for radiation at wavelengths other than where $\lambda=2\gamma_m/n$, radiation at the wavelengths $\lambda=[\lambda_{n0}-\Delta\lambda]$ also can contribute to the correlator's dc output by effectively correcting the phase of the detector output by reversing the direction of motion of the movable lamella once every $2T_m$ sec so that the output is always in phase at the $\gamma=0$ position, and is never very far out of phase with the reference signal which is $V_r(t)=V_0\cos 2\pi f_0 t$ (providing that λ is near λ_{n0}). The effect of this can be seen by plotting the correlator output as a function of time for different wavelengths of radiation near λ_{n0}. Fig. 7 shows, that for $\lambda=\lambda_{n0}\pm\Delta\lambda$, instead of a very low-frequency cosine wave having no dc component which would be obtained if the output of the interferometric modulator were modulated in a continuous manner; an output consisting of only segments of a cosine wave is obtained, and this output does have a dc component. Thus, radiation at wavelengths *near* λ_{n0} as well as radiation exactly at λ_{n0} will produce a dc output component. Frequencies which are far removed from f_{n0} (due to radiation at wavelengths not near λ_{n0}) produce rapidly varying components

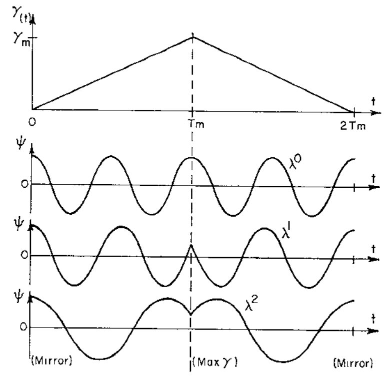

Fig. 6—The output of an interferometric modulator during one cycle.

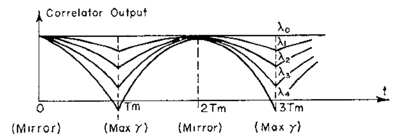

Fig. 7—The output of the correlator before integration.

which contribute little to the dc output and whose ac components can be reduced to a negligible level by passing the output of the radiation detector through a band-pass filter centered at f_{n0}. Mathematically, the equation for the modulation envelope I of the radiation power output in the wave number increment $\Delta\nu$ of the interferometric modulator as a function of ν and t where $E(\nu)$ is the input power density spectrum,[3] where $2T_m<t<[q+1]T_m$ and where $\Delta I(\nu, t+2T_m)=\Delta I(\nu, t)$, is [11]

$$\Delta I(\nu, t) = E(\nu)[\tfrac{1}{2}][1+\cos\Phi(t)][\Delta\nu] \text{ watt-cm} \quad (4)$$
$$-T_m \leq t \leq T_m$$

in which $\Phi(t)=2\pi\gamma_m\nu t/T_m$, $f_{\text{mod}}=\gamma_m\nu/T_m$ and $q=0, 2, 4, 6$, etc. Since the grating (or the interferometric mirror) moves with a period $2T_m$, the total modulator envelope output $I(t)$ will consist only of time components which are modulated at the discrete audio frequencies $f_n=n/2T_m$ where n is an integer. The magnitude of each Fourier component [*i.e.*, $I_n(t)$] of $I(t)$ can be found by finding the Fourier coefficients of the piecewise continuous (between qT_m and $[q+1]T_m$ where q is an integer) function $\{E(\nu)\cdot\frac{1}{2}\cos 2\pi\gamma_m\nu t/T_m\}$ by convolving it with the weighting function for the nth component, *i.e.*, $\frac{1}{2}\cos 2\pi\gamma_m\nu_n t/T_m$. This leads to [11]

$$I_n(t) = \left[\frac{1}{2}\int_{\nu_A}^{\nu_B} E(\nu)B_n(\nu/\nu_n)d\nu\right]\cos\frac{n\pi t}{T_m} \text{ watts} \quad (5)$$

[3] The total received power E_i is just $E(\nu)$ integrated over the bandwidth of the radiometer receiver. Similarly, $[\Delta E(\nu)\text{ min}]=[\Delta P\text{ min}]/\Delta\nu$ [see (1)].

for one Fourier component where[4]

$$B_n(\nu/\nu_n) = 2[-1]^n \frac{\sin n\pi \dfrac{\nu}{\nu_n}}{n\pi\left[\dfrac{\nu}{\nu_n} - \dfrac{\nu_n}{\nu}\right]} = \frac{\sin\{2\pi\gamma_m[\nu - \nu_n]\}}{2\pi\gamma_m[\nu - \nu_n]}. \tag{6}$$

Here ν_A to ν_B is the wave number range of the radiation entering the interferometric modulator and ν_n is the wave number at the center of the response function.[5] The magnitude of the nth component of the power variation is then given by

$$P_n = \frac{1}{2}\int_{\nu_A}^{\nu_B} E(\nu)B_n(\nu/\nu_n)d\nu \text{ watts} \tag{7}$$

where P_n is the magnitude of $I_n(t)$.

If we now let $E(\nu)$ have a broad spectrum this becomes

$$P_n = \frac{E(\nu_n)}{2}\int_{\nu_A}^{\nu_B} B_n\left(\frac{\nu}{\nu_n}\right)d\nu = \frac{E(\nu_n)}{2}\left[\frac{1}{2\gamma_m}\right] = \frac{E(\nu_n)}{4\gamma_m} \text{ watts} \tag{8}$$

when the range of ν_A to ν_B is large. From this point of view, the "bandwidth" of the radiometer in terms of the wave number ν is given as $1/\gamma_m$ which is the difference in ν between the first zero of $B_n(\nu/\nu_n)$ above ν_n and the first zero below ν_n (see Fig. 8).

Since the magnitude of the nth Fourier time component of the radiation detector output is dependent upon the magnitude of the power spectral density of the radiation input to the interferometric modulator near $\nu = \nu_n$, if we can determine the magnitude of the nth component of the detector output, then we can determine the approximate spectral power density of the radiation in input at ν_n within a bandwidth of $1/\gamma_m$. This can be done experimentally by correlating the output of the detector with a correctly phased reference signal (a cosine wave) at f_{n0} which stays in phase with the interferometric modulator drive mechanism, perhaps using a band-pass filter as a preselector (synchronous detection). The only noise component of the detector output which will produce a dc output is that at $f = f_{n0}$. Any other noise component at a frequency $f = f_{n0} \pm \Delta f$ where Δf is small will produce in the correlator output a slowly varying voltage. If Δf is more that a fraction or so of a cycle per second, the magnitude of this component in the correlator output is reduced to a negligible level by the filtering action of the correlator integrator circuit. If we consider any output except the dc output due to the signal component at $f = f_{n0}$ to be a noise contribution, we then see that the only portion of the noise in the output of the radiation detector which will produce any appreciable noise at the correlator output is that which exists in the frequency range $f_{n0} - \Delta f_p < f < f_{n0} + \Delta f_p$ where Δf_p is the one-sided pass band of the integrator circuit in the correlator. We will assume that $\Delta f_p \ll 1/4T_m$ so that the modulation frequencies f_{n0+1} and f_{n0-1} adjoining f_{n0} will not produce any appreciable noise contribution to the output signal. If this is not true, a periodic type of noise signal will appear on the recorded output of the correlator.

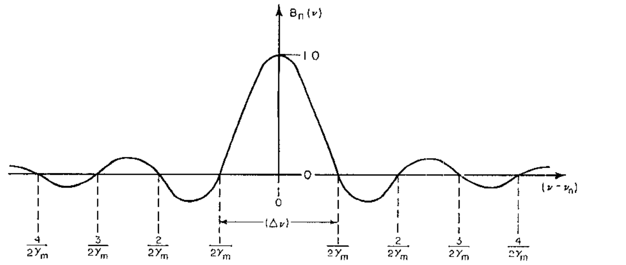

Fig. 8—Response function of the submillimeter interferometric radiometer. The interval $(\Delta\nu)$ is taken to be the "bandwidth" of the radiometer.

The Minimum Detectable Spectral Density of a Submillimeter Interferometric Receiver

For a relatively smooth input density spectrum $E(\nu)$ the component modulated at f_{n0} will be $I_n(t) = E(\nu)[\cos \pi nt/T_m]/4\gamma_m$. This produces a dc output component whose magnitude is $E(\nu_n)[G'V_0]/8\gamma_m$ where V_0 is the amplitude of the reference signal $V_r(t) = V_0 \cos \pi nt/T_m$ and G' is a gain constant including the detector constant, amplification factor and correlator constant.

If the rms noise fluctuations referred to the input power of the radiation detector have a spectrum given by $P(f)$ (per unit of audio-frequency bandwidth) then the rms noise voltage spectrum contributed to the output is approximately $P(f)G'V_0[\Delta f_p]$. If we now let this equal the dc signal voltage the mininum detectable change in the spectral density of the radiation signal becomes

$$[\Delta E(\nu_n) \min] = 8\gamma_m P\left(f_{n0} = \frac{\gamma_m}{T_m}\nu_{n0}\right)[\Delta f]_p \text{ watt-cm.} \tag{9}$$

[4] Here the term

$$\frac{\sin\{2\pi\gamma_m[\nu + \nu_n]\}}{2\pi\gamma_m[\nu + \nu_n]}$$

has been dropped since it contributes little when integrated from ν_A to ν_B as compared with the term

$$\frac{\sin\{2\pi\gamma_m[\nu - \nu_n]\}}{2\pi\gamma_m[\nu - \nu_n]}$$

where ν_n lies between ν_A and ν_B.

[5] Also, another method has recently been proposed to give the interferometric modulator a more desirable response function than the sin X/X form discussed in this paper. Details are found in Williams [24] and will be published after experimental verification is obtained.

This can also be written as

$$[\Delta E(\nu_n) \min] = 8\gamma_m P(f_{n0} = v\nu_{n0})[\Delta f]_p \text{ watt-cm.} \quad (10)$$

In analyzing this result we must remember that the frequency of modulation is related to the wave number by the relation $f = v\nu$ where $v = d\gamma/dt$. This means that if we are interested in scanning a range of radiation having wave numbers from ν_1 to ν_2 we must vary the path length at velocities from $v_1 = f_{n0}/\nu_1$ to $v_2 = f_{n0}/\nu_2$ where f_{n0} is the frequency of operation of the synchronous detector (or the reference frequency). To achieve the best result it would be desirable to select f_{n0} such that it falls at the point where the detector and electronic noises have their lowest spectral densities and where the construction and operation of the electronics circuitry is convenient.

Once f_{n0} is determined, v is determined by the relation $v = f_{n0}/\nu_{n0}$ where ν_{n0} is the central wave number of the radiation which we desire to receive. Then the range of wave numbers (*i.e.*, the bandwidth) desired determines γ_m. For example, the first minimum in $B_n(\nu/\nu_n)$ occurs at

$$\gamma_m[\nu_m - \nu_n] = \pm \tfrac{1}{2} \quad (11)$$

$$\gamma_m = \pm \frac{1}{2[\nu_m - \nu_n]} \text{ cm} \quad (12)$$

where ν_m is the point of the first minimum. With γ_m thus determined, T_m is determined by

$$T_m = \frac{\gamma_m}{v} \text{ sec.} \quad (13)$$

The practical considerations of electronically filtering out the adjacent components of f_{n0} mentioned previously determine an upper limit on T_m but do not keep us from making it as small as we desire. To obtain the highest sensitivity we should make T_m and γ_m as small as possible which causes the response curve to spread out along the wave number axis.

Radiation Detectors for Use With an Interferometric Radiometer Receiver

Since the minimum detectable temperature is proportional to $P(f)$ it is necessary to discuss the various types of detectors suitable for detecting submillimeter-wavelength radiation and the sources of noise in such detection systems. There are two general types of noises which are important to radiometers. The first type of noise may be divided into two parts: the noise which accompanies the signal at the output of the radiation detector and the electronic noise, hum and stray-signal pickup introduced by the electronic equipment. The second type of noise, that due to gain fluctuations, may be caused by variations in one of a number of other parameters which affect the over-all gain of the system.

The detector noise will depend upon the type of radiation detector being used. It may result from variations in the temperature of the surroundings of the detector, stray radiation power which leaks into the detector, Brownian motion of the gas or vibration effects (in the case of pneumatic detectors), bias current shifts (in bolometric detectors) and so forth. Consider now the equivalent input power spectral density $P_D(f_c)$ of the detector at some modulation or chopping frequency f_c and assume that the bandwidth Δf_p involved is equal to the reciprocal of the time constant τ of the correlator integrator circuit. Then, a quantity called the noise-equivalent power (NEP) is defined as

$$\text{NEP} = \frac{P_D(f_c)}{\tau} \text{ watts.} \quad (14)$$

The NEP for several types of submillimeter detectors is tabulated in Table I, along with the τ and f_c at which the measurements were made. Of the detectors listed in Table I the Golay cell is probably the most commonly used, but it is extremely sensitive to vibration noise and to erratic fluctuations due to stray air currents. Noise in the electronic system due to hum, stray pickup, thermal noise, etc., in the amplifier may be treated in a manner similar to detector noise and may be expressed as an equivalent noise power spectral density $P_E(f)$ referred to the detector input.

TABLE I
CHARACTERISTICS OF SUBMILLIMETER-WAVELENGTH DETECTORS

Detector	NEP	Time Constant (τ)	Chopping Frequency (f)	References
Superconducting Bolometer	10^{-12}	0.6 sec	10 cps	12
Golay Cell	10^{-10}	10.0 sec	10–13	13
Semiconductor Bolometer	10^{-11} 5×10^{-13}	1.0 sec 1.0 sec	— —	14 15
Photoconductive Detector	5×10^{-11}	1.0 sec	—	16, 17

The noise sources discussed thus far cause serious noise effects in the recorded output only if they have appreciable spectral densities in the region near the audio reference frequency f_{n0}. The noise introduced by gain fluctuations is more widespread in that the correlation processes do not reduce its effects upon the output of the correlator. The gain-fluctuations noise is due to fluctuations in the amplifier gain, the Golay cell exciter-lamp voltage, the bolometer bias current, the correlator reference-signal level, etc., all of which serve to change the total effective ac gain of the system.

If the total gain of the system is G[6] and the fluctuation in the gain is given by ΔG, then the dc output voltage for a total effective power input E_i[7] is given by $Y_0 = GE_i$.

[6] G, the total gain, is given by the ratio of the correlator dc output to the rms amplitude of the radiation power variations at the detector input. $G = G_1, G_2, G_3$ where these are the gains of the radiation detector, the amplifier and the correlator, respectively.

[7] E_i is the total radiation power input to the modulator [*i.e.*, $E(\nu)$ integrated over the bandwidth ($\Delta\nu$) of the receiver].

The gain fluctuation ΔG will give an output voltage fluctuation $\Delta Y_0 = [\Delta G] E_i$. This is approximately equivalent to an apparent shift in the total input power of

$$\Delta E_i = \frac{\Delta Y_0}{G} = \frac{\Delta G}{G} E_i \text{ watts} \tag{15}$$

or

$$\frac{\Delta E_i}{E_i} = \frac{\Delta G}{G}. \tag{16}$$

If we let ΔG be the rms value of the gain fluctuations, then ΔE_i, the equivalent indeterminancy of the radiation power level, depends upon the magnitude of the radiation power being measured.

The quantities $P_D(f)$, $P_E(f)$ and $\Delta E(\nu) = [\Delta G/G] E(\nu)$ are all expressed as rms fluctuations in the power density spectrum and are of a random nature. Therefore, in order to find the total rms noise fluctuations given in terms of $P(f)$ we must take the root of the sum of the squares of the above quantities,

$$P_{(f)} = \left[P_D^2(f) + P_E^2(f) + \left[\frac{\sqrt{2}\, v}{\pi} \left\{ \frac{\Delta G}{G} \right\} E\left(\nu = \frac{f}{v} \right) \right]^2 \right]^{1/2} \text{ watt-sec.} \tag{17}$$

This is the $P(f)$ used in (10). Since we must assume that the antenna of a radiometer is looking at some background temperature, and therefore, that the radiometer receiver has a considerable amount of radiation power being delivered to it, we can not neglect the last term (*i.e.*, in radiometer studies we are interested in determining the minimum temperature difference which a target must have to be distinguishable from its background). The gain fluctuation therefore may be an important factor in determining the performance of the radiometer.

The Effects of Atmospheric Water Vapor Absorption

One of the problems of radiometry in the far infrared and submillimeter regions is the absorption of radiation in this region by atmospheric water vapor. This problem is especially acute if a ground-based radiometer is to be used at a low-altitude, high-humidity location. This problem may be partially overcome by making use of the places in the spectrum where the absorption is the weakest. These are known as atmospheric "windows."

Usually these spectral windows are not very wide (*i.e.*, do not cover much of the spectrum) and are not very transparent. Very strong absorption would completely preclude the possibility of using a ground-based radiometer to measure radiation from a source outside the atmosphere. Moreover, any attenuation placed in the path of a signal will also generate thermal noise of its own. Thus, the atmospheric water vapor will also act as a wide-band source of noise whose emission wavelengths coincide with the wavelengths of the absorption bands. Therefore, in designing a radiometer it is desirable to have its bandwidth nearly the same as the bandwidth of the spectral window being used, both to receive the maximum signal and to reduce the noise level.

Several authors have published data on atmospheric water vapor absorption over relatively short path lengths in the submillimeter region. Independent measurements by Genzel [11] and by Rowntree, *et al.* [13], permit us to determine the position of some atmospheric windows and to obtain a rough estimate of the width of these windows to use in determining the optimum bandwidth of the interferometric receiver. Based upon these estimates we have tentatively selected the 28.5 cm^{-1} (*i.e.*, 855 Gc) region for the design of our radiometer and have chosen the bandwidth to be 2.2 cm^{-1} (66 Gc). Fig. 9 shows this atmospheric window.

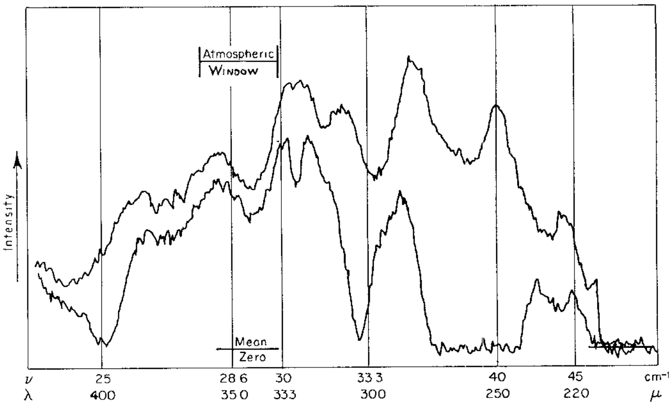

Fig. 9—The water vapor absorption data shown here was supplied through the courtesy of Sanderson, Department of Physics, The Ohio State University, using the far infrared spectrometer built by Vance, *et al.* [17]. The lower trace was made at atmospheric pressure at 75° F and at 51 per cent humidity. The upper trace was made with the instrument evacuated. The difference between the two traces indicates the relative water vapor absorption. The path length involved was 11 meters.

Estimation of the Performance of a Typical Submillimeter Radiometer

Based upon the preceding theory, a submillimeter radiometer was designed (see Figs. 10 and 11) and its characteristics calculated [1]. The various design parameters were $F_r = 4.4$, $\delta_2 = 1.0$ m,[8] $A_m = 10$ m^2, $F_0 = 4.4$, $\Delta\Omega = 10^{-6}$ sterad (0.06° beamwidth). The radiometer was designed for operation at the 28.5 cm^{-1} atmospheric window with a half bandwidth of 1.1 cm^{-1} wave numbers. This required that $D = 3.5$ m, $\delta_1 = 15.4$ m, $s = 2.36$ cm^2, $v = 4.55$ mm/sec, $\gamma_m = 4.55$ mm and $T_m = 1$ sec. The integration time constant was chosen at 5 sec and it was assumed that the radiation detector would have a

[8] δ_2 = focal length of receiver optics.

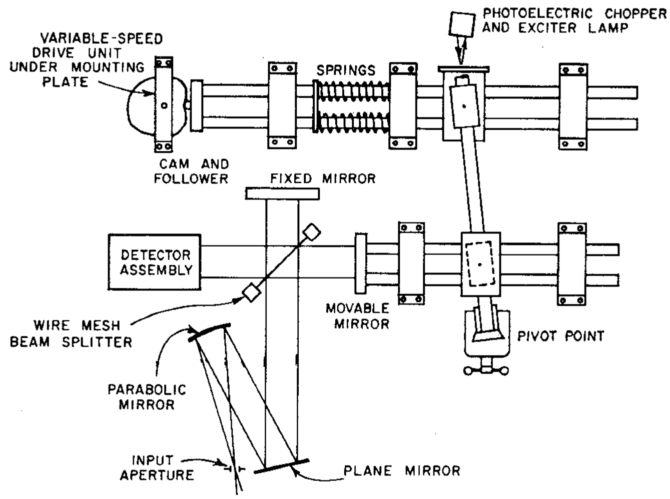

Fig. 10—Diagram of the submillimeter-interferometric radiometer. The reciprocal motion of the movable mirror is provided by a cam-and-follower arrangement operating through a ratio arm. The magnitude of the mirror movement is controlled by the position of the pivot point which is adjustable by means of a micrometer screw. The photoelectric chopper provides the reference signal for the correlator, and the detector assembly contains the radiation detector and its optical system. The input aperture is at the focus of a Cassegrain collection system which serves as the antenna.

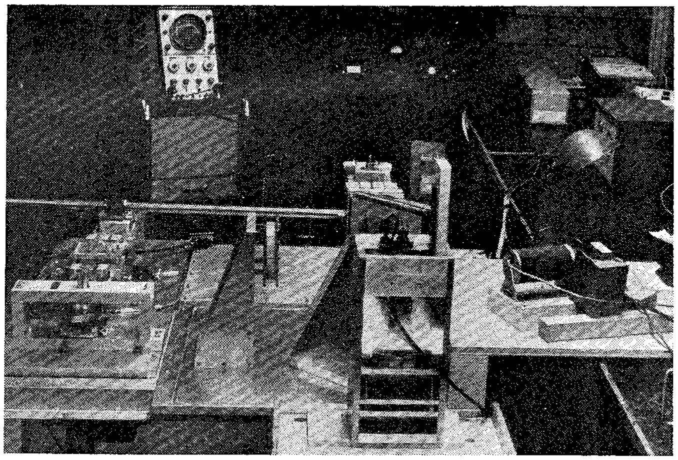

Fig. 11—A general view of the interferometric modulator portion of the submillimeter radiometer which is presently under construction. The cam-and-follower mechanism is on the left and the radiation detector (Golay cell) assembly is in the foreground. To the rear of the detector the top of the beam splitter is visible, to the left of this the fixed mirror and farther to the rear the supports for the movable mirror. To the right is a black-body source and mercury arc lamp for alignment and test purposes.

sensitivity of 10^{-11} watts with a 1 sec integration time and that gain-fluctuation effects were negligible. With these parameters the estimated minimum detectable temperature difference, not taking into account the atmospheric effects, was 0.23° K with a 300° K background temperature. This probably represents the best results obtainable with a 10 ft antenna and a 0.06° beamwidth. With an average Golay cell used as the radiation detector, the minimum detectable temperature change would be about 23° K using the same antenna and beamwidth. Decreasing the antenna diameter would have no effect upon $[\Delta T_{\min}]$ as long as F_0 and s remained the same and as long as the source completely filled the beamwidth. However, as the antenna becomes smaller the beamwidth becomes larger due to a shorter focal length. For a 12 inch diameter antenna it is about 0.6°. If s would be decreased to 0.236 cm^2 so as to maintain the 0.06° beamwidth, $[\Delta T_{\min}]$ would be about 2.3° K for the sensitive detector and 230° K for the Golay cell, using a 5 sec integration time with both detectors.

Experimental Results

The Physics Department [13], [17]–[19], The Ohio State University, has a far infrared spectrometer in which a grating type interferometric modulator is employed to reject radiation occurring in the higher-order diffraction modes from a conventional echelette grating monochromator. It was arranged to use just the interferometric modulator and detector portions of this instrument in an attempt to check the validity of the theory developed in this paper. Using this theory it was calculated that for the particular instrument involved (which used a Golay cell radiation detector) that the minimum detectable change in the temperature of a black-body source placed at the input aperture should be about 7° K at 465 micron wavelength radiation, 15° K at 611 microns and 63° K at $\lambda = 1$ mm. The experimentally detectable temperature changes, using a Barnes Engineering model 11-101 black-body source were 7.4 to 13.5° K at 465 microns, 26.6° K at 611 microns, and 285° K at $\lambda = 1$ mm. It is thought that the fairly large disagreement between the measured and the calculated results at the 1 mm wavelength may be due to deviation of the black-body source from the ideal radiation law.

Conclusions

In the foregoing we have seen how the field of submillimeter radiometry borrows from the theory and techniques of both radio astronomy and visible light astronomy, and also embraces a great number of problems peculiar to this wavelength region. The power emitted by most black bodies in this region of wavelengths is already much weaker than in the visible light region, while the sensitive receivers used in the centimeter-wavelength region are not as yet available at the submillimeter wavelengths. This necessitates the use of broad-band receivers using quasi-optical techniques, which, in general, have poor sensitivities when compared to the superheterodyne receivers used in the microwave region. We have also seen that the presence of strong water vapor absorption in the submillimeter region further limits the sensitivity of submillimeter radiometers and may present a major obstacle to conducting ground-based radiometry studies and will probably necessitate locating the instrument above the major portion of the earth's atmosphere on top of a mountain, in a balloon, in a high flying aircraft, or in a satellite.

In this paper we have mentioned a few of the problems connected with submillimeter radiometry and have proposed and analyzed a particular type of submillimeter radiometer which we call the interferometric radiometer (since it is based upon the principles of operation of the interferometric spectrometer). This type of radiometer, we believe, represents the best than can be devised at the present time with the existing techniques which can be employed at submillimeter wavelengths. A great increase in the sensitivity of submillimeter radiometers over the type proposed here could be effected if and when the coherent techniques used in the radio and microwave region could be extended to the submillimeter region.

Appendix

Comparison of the Minimum Detectable Power Spectral Density in Centimeter and Submillimeter Radiometers

It is interesting to compare the equation developed for the minimum detectable power spectral density of the periodic interferometric radiometer receiver,

$$[E(\nu_n)\ \min] = 8\gamma_m P(f_{n0})[\Delta f]_p \text{ watt-cm}, \tag{18}$$

with that for the minimum power change detectable by Dicke radiometer [20]–[22],

$$[\Delta P \min] = K\left[\frac{[F_m - 1]}{\sqrt{B_{IF}\,\tau}}\, N + \left[\frac{G(t) - G(0)}{G(0)}\right]\cdot[\Delta N + N_s]\right] \text{ watts.} \tag{19}$$

In (19) $[\Delta P \min]$ is the minimum detectable power, all of which must be received in a bandwidth B_{IF}; and $[F_m - 1]\ N_0$ is the total noise power of the receiver within this bandwidth, where F_m is the receiver noise factor, N_0 is the ambient temperature noise power, τ is the integrator time constant, $G(t)$ is the gain of the system as a function of time, $G(0)$ is the time-averaged gain, K is a factor accounting for detector and recorder noise, N_s is the noise power received from the source, and ΔN is the change in the total noise power. We now wish to compare the *first* part of (19) with (18). To facilitate this comparison we shall call the pass band of the interferometric radiometer by the symbol $B_{IF}(\nu)$ [the pass band can be expressed in terms of wave numbers (cm^{-1}) as well as in terms of the frequency (cps)]. Then the minimum power detected by the interferometric radiometer in the pass band $B_{IF}(\nu)$ will be

$$[\Delta P_{\min}] = 8\gamma_m P(f)[\Delta f]_p B_{IF}(\nu) \text{ watts.} \tag{20}$$

This does not bear much resemblance to (19), but as we shall show, the two are very closely related. First consider $P(f)$. This is the spectral density of the rms fluctuations in the noise power level at the input of the radiation detector, and is related to the total noise power spectral density by the proportionality

$$P(f) \propto \sqrt{B_{IF}(\nu)\tau}\ P(\nu) \text{ watt-sec} \tag{21}$$

by a process somewhat analogous to that used by Dicke [20]. We will call the product $B_{IF}(\nu)P(\nu)$ by the symbol N_0'. Also, since γ_m is proportional to the reciprocal of the spectral pass band and $[\Delta f]_p$ is proportional to $1/\tau$, (20) becomes

$$[\Delta P_{\min}] \propto \left[\frac{1}{B_{IF}(\nu)}\right][\sqrt{B_{IF}(\nu)\tau}]\left[\frac{N_0'}{B_{IF}(\nu)}\right]\cdot\left[\frac{1}{\tau}\right][B_{IF}(\nu)] \text{ watts.} \tag{22}$$

When the common terms are cancelled this becomes

$$[\Delta P_{\min}] \propto \frac{N_0'}{\sqrt{B_{IF}(\nu)\tau}} \text{ watts} \tag{23}$$

which, except for constant and unit-conversion factors, is very similar to the first part of (19). This serves to make the results obtained from the analysis of the submillimeter-interferometric radiometer compatible with the results which have been obtained for microwave radiometers.

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Some Considerations in the Design of Narrow-Band Waveguide Filters*

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Summary—**It is natural that design considerations common to all kinds of filters should have received most attention in the literature. Some design considerations which are peculiar to waveguide filters (being due to the dispersive property of waveguides) are treated in this paper. It is shown that there are waveguide dimensions which minimize the filter dissipation loss and also the filter pulse power capacitance and which keep the nearest spurious-frequency response farthest from the fundamental pass band. Graphical data are presented to show how much is to be gained or lost by departing from the usual dimensions.**

Introduction

A FILTER (by definition) is required to operate selectively over a spectrum of frequencies. The specified frequency behavior can usually be met by a variety of possible designs, some of which are better suited than others to satisfy various additional requirements which may be specified or desirable; the electrical requirements may include low dissipation loss, high pulse power capacitance and freedom from spurious responses, in addition to the specified amplitude or phase characteristics.

Some general design considerations, common to lumped-constant, TEM-line (nondispersive) and waveguide (homogeneously dispersive) filters will first be summarized. Then we will discuss some special considerations that are peculiar to waveguide filters, which have an extra degree of freedom allowed by the choice of waveguide cutoff wavelength.

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General Considerations for Band-Pass Filters

The design of narrow-band filters can be based on a lumped-constant, low-pass prototype circuit.[1] Formulas and tables[2] exist for filter elements g_i, which give maximally flat, Chebyshev and maximally flat time delay characteristics. Sometimes it is preferred to make the filter elements all equal to one another (all $g_i = g$). A microwave filter based on this prototype may be considered to consist of a cascade of identical cavities and will be called a matched-periodic (or just periodic) filter.[3,4] S. B. Cohn[5,6] has shown that such a filter has

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